CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/33 Paper 3, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2014 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

CAMBRIDGE International Examinations

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

1	Either	State or imply non-modular inequality $(3x-1)^2 < (2x+5)^2$ or corresponding quadratic equation or pair of linear equations $3x-1 = \pm(2x+5)$	B1	
		Solve a three-term quadratic or two linear equations $5x^2 - 26x - 24 < 0$	M1	
		Obtain $-\frac{4}{5}$ and 6	A1	
		State $-\frac{4}{5} < x < 6$	A1	
	Or	Obtain value 6 from graph, inspection or solving linear equation	B1	
		Obtain value $-\frac{4}{5}$ similarly	B2	
		State $-\frac{4}{5} < x < 6$	B1	[4]

2	Use correct product rule or correct chain rule to differentiate y	M1	
	Use $\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$	M*1	
	Obtain $\frac{-4\cos\theta\sin^2\theta + 2\cos^3\theta}{\sec^2\theta}$ or equivalent	A1	
	Express $\frac{dy}{dx}$ in terms of $\cos\theta$	DM*1	
	Confirm given answer $6\cos^5\theta - 4\cos^3\theta$ legitimately	A1	[5]

3	**(i)**	Either	Equate p(−1) or p(−2) to zero or divide by $(x+1)$ or $(x+2)$ and equate constant remainder to zero.	M*1	
			Obtain two equations $a - b = 6$ and $4a - 2b = 34$ or equivalents	A1	
			Solve pair of equations for a or b	DM*1	
			Obtain $a = 11$ and $b = 5$	A1	
		Or	State or imply third factor is $4x-1$	B1	
			Carry out complete expansion of $(x+1)(x+2)(4x-1)$ or $(x+1)(x+2)(Cx+D)$	M1	
			Obtain $a = 11$	A1	
			Obtain $b = 5$	A1	[4]
	(ii)		Use division or equivalent and obtaining linear remainder	M1	
			Obtain quotient $4x + a$, following their value of a	A1√	
			Indicate remainder $x - 13$	A1	[3]

4	**(i)**	Either Use $\cos(A \pm B)$ correctly at least once	M1	
		State correct complete expansion	A1	
		Confirm given answer $\cos\theta$ with explicit use of $\cos 60^o = \frac{1}{2}$	A1	
		SR: "correct" answer from sign errors in both expansions is B1 only		
		Or Use correct $\cos A + \cos B$ formula	M1	
		State correct result e.g. $2\cos\left(\frac{2\theta}{2}\right)\cos\left(\frac{-120}{2}\right)$	A1	
		Confirm given answer $\cos\theta$ with explicit use of $\cos(\pm 60^o) = \frac{1}{2}$	A1	[3]
	(ii)	State or imply $\frac{\cos 2x}{\cos x} = 3$	B1	
		Obtain equation $2\cos^2 x - 3\cos x - 1 = 0$	B1	
		Solve a three-term quadratic equation for $\cos x$	M1	
		Obtain $\frac{1}{4}\left(3 - \sqrt{17}\right)$ or exact equivalent and, finally, no other	A1	[4]
5	**(i)**	State or imply $\mathrm{i}w = -3 + 5\mathrm{i}$	B1	
		Carry out multiplication by $\frac{4-\mathrm{i}}{4-\mathrm{i}}$	M1	
		Obtain final answer $-\frac{7}{17} + \frac{23}{17}\mathrm{i}$ or equivalent	A1	[3]
	(ii)	Multiply w by z to obtain $17 + 17\mathrm{i}$	B1	
		State $\arg w = \tan^{-1}\frac{3}{5}$ or $\arg z = \tan^{-1}\frac{1}{4}$	B1	
		State $\arg wz = \arg w + \arg z$	M1	
		Confirm given result $\tan^{-1}\frac{3}{5} + \tan^{-1}\frac{1}{4} = \frac{1}{4}\pi$ legitimately	A1	[4]
6	**(i)**	State or imply correct ordinates 1, 0.94259…, 0.79719…, 0.62000…	B1	
		Use correct formula or equivalent with $h = 0.1$ and four y values	M1	
		Obtain 0.255 with no errors seen	A1	[3]
	(ii)	Obtain or imply $a = -6$	B1	
		Obtain x^4 term including correct attempt at coefficient	M1	
		Obtain or imply $b = 27$	A1	
		Either Integrate to obtain $x - 2x^3 + \frac{27}{5}x^5$, following their values of a and b	B1$\sqrt{}$	
		Obtain 0.259	B1	
		Or Use correct trapezium rule with at least 3 ordinates	M1	
		Obtain 0.259 (from 4)	A1	[5]

7	**(i)**	State at least two of the equations $1+\lambda = a+\mu$, $4 = 2+2\mu$, $-2+3\lambda = -2+3a\mu$	B1	
		Solve for λ or for μ	M1	
		Obtain $\lambda = a$ (or $\lambda = a+\mu-1$) and $\mu = 1$	A1	
		Confirm values satisfy third equation	A1	[4]
	(ii)	State or imply point of intersection is $(a+1, 4, 3a-2)$	B1	
		Use correct method for the modulus of the position vector and equate to 9, following their point of intersection	M*1	
		Solve a three-term quadratic equation in a $\quad (a^2 - a - 6 = 0)$	DM*1	
		Obtain –2 and 3	A1	[4]
8	**(i)**	Sensibly separate variables and attempt integration of at least one side	M1	
		Obtain $2y^{\frac{1}{2}} = \ldots$ or equivalent	A1	
		Correct integration by parts of $x\sin\frac{1}{3}x$ as far as $ax\cos\frac{1}{3}x \pm \int b\cos\frac{1}{3}x\,\mathrm{d}x$	M1	
		Obtain $-3x\cos\frac{1}{3}x + \int 3\cos\frac{1}{3}x\,\mathrm{d}x$ or equivalent	A1	
		Obtain $-3x\cos\frac{1}{3}x + 9\sin\frac{1}{3}x$ or equivalent	A1	
		Obtain $y = \left(-\frac{3}{10}x\cos\frac{1}{3}x + \frac{9}{10}\sin\frac{1}{3}x + c\right)^2$ or equivalent	A1	[6]
	(ii)	Use $x = 0$ and $y = 100$ to find constant	M*1	
		Substitute 25 and calculate value of y	DM*1	
		Obtain 203	A1	[3]
9	**(i)**	Sketch increasing curve with correct curvature passing through origin, for $x \geq 0$	B1	
		Recognisable sketch of $y = 40 - x^3$, with equation stated, for $x > 0$	B1	
		Indicate in some way the one intersection, dependent on both curves being roughly correct and both existing for some $x < 0$	B1	[3]
	(ii)	Consider signs of $x^3 + \ln(x+1) - 40$ at 3 and 4 or equivalent or compare values of relevant expressions for $x = 3$ and $x = 4$	M1	
		Complete argument correctly with correct calculations (–11.6 and 25.6)	A1	[2]
	(iii)	Use the iterative formula correctly at least once	M1	
		Obtain final answer 3.377	A1	
		Show sufficient iterations to justify accuracy to 3 d.p. or show sign change in interval (3.3765, 3.3775)	A1	[3]
	(iv)	Attempt value of $\ln(x+1)$	M1	
		Obtain 1.48	A1	[2]

10	State or imply $\dfrac{du}{dx} = e^x$	B1	
	Substitute throughout for x and dx	M1	
	Obtain $\displaystyle\int \frac{u}{u^2 + 3u + 2}\,du$ or equivalent (ignoring limits so far)	A1	
	State or imply partial fractions of form $\dfrac{A}{u+2} + \dfrac{B}{u+1}$, following their integrand	B1	
	Carry out a correct process to find at least one constant for their integrand	M1	
	Obtain correct $\dfrac{2}{u+2} - \dfrac{1}{u+1}$	A1	
	Integrate to obtain $a\ln(u+2) + b\ln(u+1)$	M1	
	Obtain $2\ln(u+2) - \ln(u+1)$ or equivalent, follow their A and B	A1√	
	Apply appropriate limits and use at least one logarithm property correctly	M1	
	Obtain given answer $\ln\dfrac{8}{5}$ legitimately	A1	[10]
	SR for integrand $\dfrac{u^2}{u(u+1)(u+2)}$		
	State or imply partial fractions of form $\dfrac{A}{u} + \dfrac{B}{u+1} + \dfrac{C}{u+2}$	(B1)	
	Carry out a correct process to find at least one constant	(M1)	
	Obtain correct $\dfrac{2}{u+2} - \dfrac{1}{u+1}$	(A1)	
	…complete as above.		